FAR BEYOND

MAT122

Average Change

Stony Brook University

Accumulated Change

ex. Based on the graph below, which salesperson has the most total sales after 6 months?

Salesperson A (because their y-values are consistently higher over 6 months)

Which salesperson has the most total sales after the first year? What are sales for each?

Count boxes under each graph:

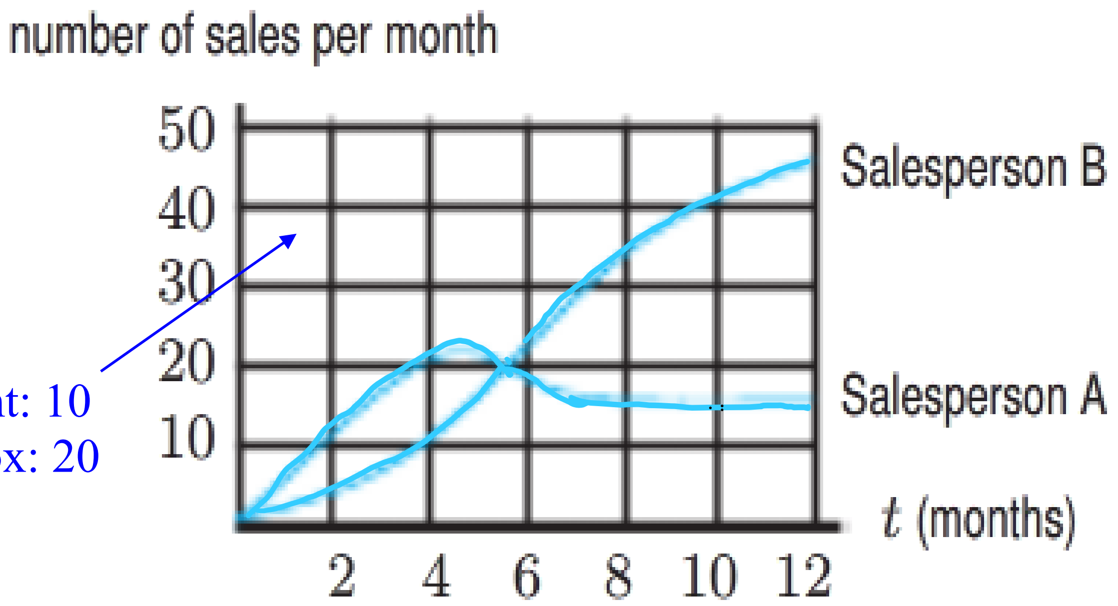
Salesperson A:

~8.75 boxes 8.75 (20) = $175 in sales

Salesperson B:

~13.25 boxes 13.25 (20) = $265 in sales

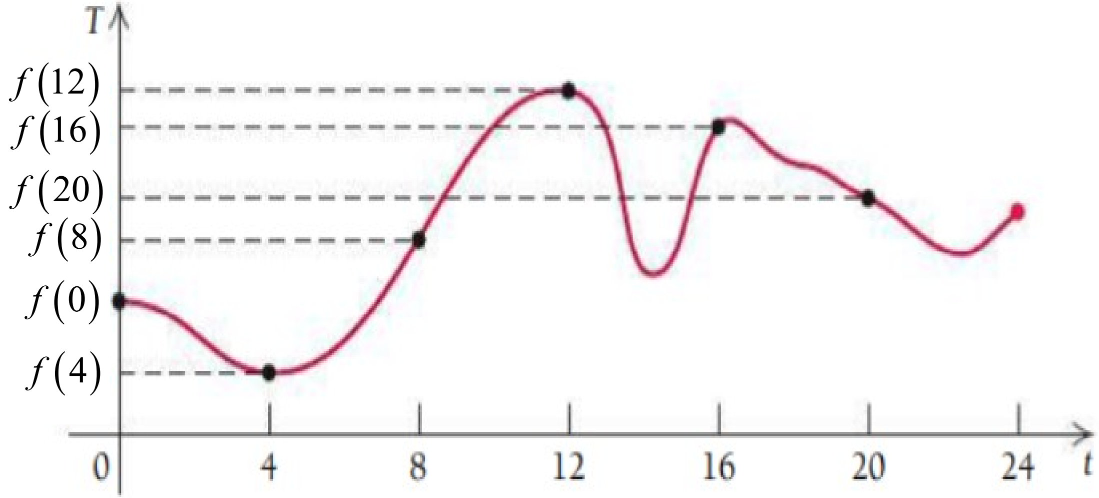
base: 2 height: 10
area of each box: 20



Average Value of a Continuous Function - Intro

An important use of area under the curve is finding the **average value** of a <u>continuous</u> function over a <u>closed</u> interval.

ex. Suppose a weather station measures the temperature (T) at time t over a 24-hour period, $[0, 24]$.
The function is continuous:



To find average temperature for the day, might take 6 temperature readings at 4-hr intervals, starting at midnight.

Overall average temperature could then be <u>estimated</u> by adding the recorded values and dividing by 6.

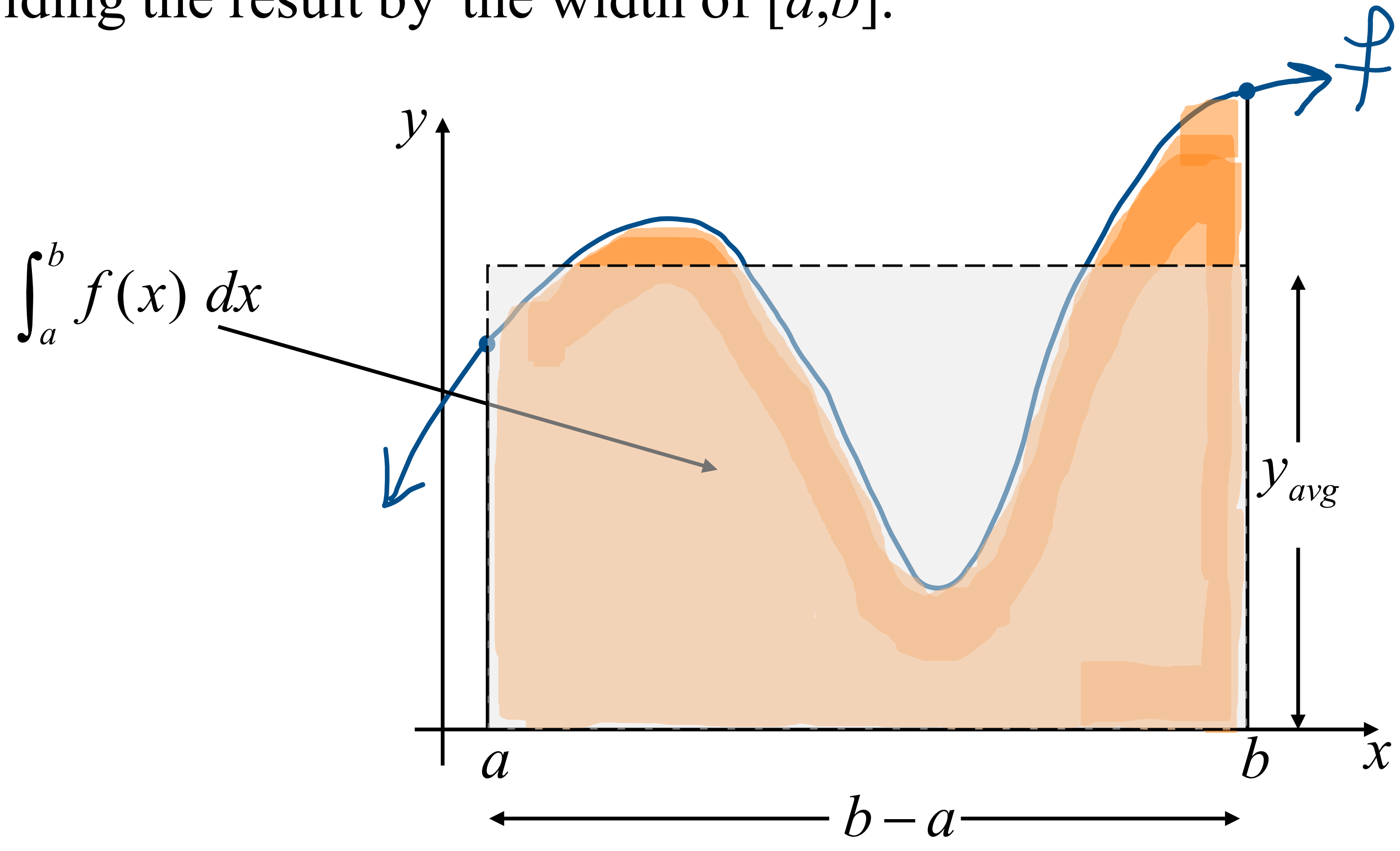
$$T_{avg} \approx \frac{f(0)+f(4)+f(8)+f(12)+f(16)+f(20)}{6}$$

As can be expected, more readings would yield a more accurate estimate.

Average Value of a Continuous Function - Exact

Finding exact average value is simply done by evaluating the definite integral over $[a,b]$ and dividing the result by the width of $[a,b]$.

closed interval



Average Value of a Continuous Function

Let f be continuous over $[a,b]$. Then its **average value**, y_{avg}, over $[a,b]$ is given by:

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

ex. Find the average value of $f(x) = x^2$ over $[0,2]$.

$$y_{avg} = \frac{1}{2-0}\int_0^2 x^2 dx$$

$$= \frac{1}{2} \cdot \frac{x^3}{3}\Bigg|_0^2$$

$$= \frac{1}{2}\left(\frac{2^3}{3} - 0\right)$$

$$= \frac{1}{2}\left(\frac{8}{3}\right) = \boxed{\frac{4}{3}}$$

Average Value – Example #2

ex. Rico's speed, in mph, t min after entering the freeway, is given by:

$$y_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$v(t) = -\frac{1}{200}t^3 + \frac{3}{20}t^2 - \frac{3}{8}t + 60 \qquad t \le 30$$

From 5 min after entering freeway to 25 min after doing so, what is Rico's average speed?

$$y_{avg} = \frac{1}{25-5}\int_5^{25}\left(-\frac{1}{200}t^3 + \frac{3}{20}t^2 - \frac{3}{8}t + 60\right)dt$$

$$= \frac{1}{20}\left(-\frac{1}{200}\frac{t^4}{4} + \frac{\cancel{3}}{20}\frac{t^3}{\cancel{3}} - \frac{3}{8}\frac{t^2}{2} + 60t\right)\Bigg|_5^{25}$$

$$= \frac{1}{20}\left(-\frac{1}{800}t^4 + \frac{1}{20}t^3 - \frac{3}{16}t^2 + 60t\right)\Bigg|_5^{25} = F(b) - F(a)$$

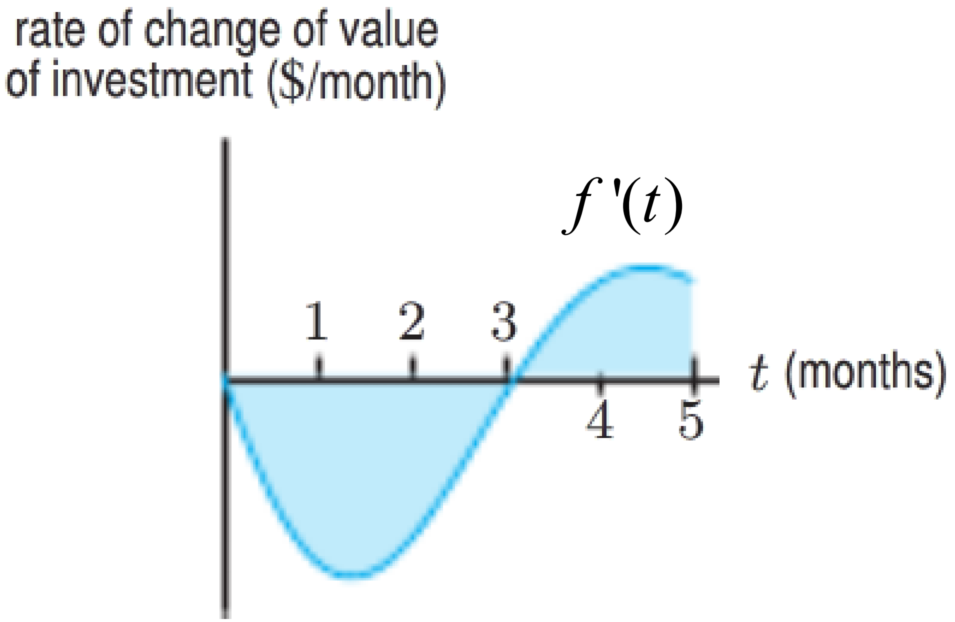
$$= \frac{1}{20}\left(-\frac{1}{800}25^4 + \frac{1}{20}25^3 - \frac{3}{16}25^2 + 60\cdot 25\right) - \left(-\frac{1}{800}5^4 + \frac{1}{20}5^3 - \frac{3}{16}5^2 + 60\cdot 5\right)$$

$$= \frac{1}{20}\left(\frac{53{,}625}{32} - \frac{9625}{32}\right) = \frac{44{,}00\cancel{0}}{2\cancel{0}\cdot 32} = \frac{2200}{32} = \frac{22\cdot 100}{4\cdot 8} = \frac{\cancel{2}\cdot 11\cdot 25\cdot \cancel{4}}{4\cdot \cancel{8}} = \frac{275}{4} = \boxed{68\frac{3}{4}\text{ mph}}$$

Change in Value

ex. $f(t)$ is the value of an investment over a 5-month period

$f'(t)$ is the rate of change of $f(t)$



a. When is the value increasing and decreasing?

Decreasing first 3 months; increasing last 2 months.

f' negative f' positive

b. After 5 months, has the value increased or decreased?

Total change in value, f, is *net* area under curve of $f' = \int_0^5 f'(t)\,dt$

Answer: over 5 months, value has decreased.